Mr. P. Siva Sankar
M.Tech (SE)
Asst. Prof.

Unit - 4 (part - A)

Laminar flow :—

The Flow of a fluid when each particle of the fluid follows a smooth path, paths which never interfere with one another.

Turbulent flow :—

It is an irregular flow that is characterized by tiny Whirlpool regions.

Characteristics of Turbulent Flow :—

→ Turbulent flow tends to occur at Higher velocities, low viscosity and at Higher characteristic linear dimensions.

→ If the Reynolds number is greater than $Re > 3500$, the flow is Turbulent.

→ The Flow is characterized by irregular movement of fluid.

→ The process of turbulence is highly unsteady, so that the flow velocity at a given point is show to great variance over time.

→ Turbulence is a three dimensional phenomenon

Characterstics of Laminar flow :—

→ It is characterized by smooth stream lines and highly ordered motion.

If the Reynolds number is less than $Re > 2300$, the flow is Laminar

Laminar flow is common only in cases in which the flow channel is relatively small, the fluid is moving slowly and its viscosity is relatively High.

→ there is no motion in radial direction and the velocity component in the direction normal to flow is zero.

→ There is no accerlation since the flow is steady.

→ The steady laminar flow of an Incompressible fluid with Constant properties in the fully developed region of a straight Circular pipe.

Laws of Friction :—

there are mainly 5 laws

~~Whean on ob~~

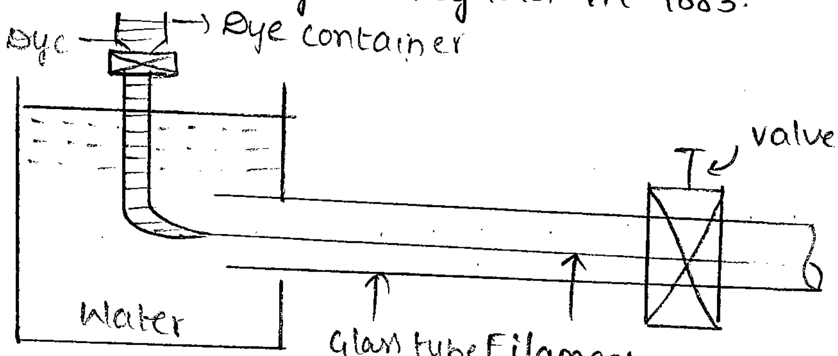
Laminar Flow	Turbulent flow
* It is proportional to velocity of Flow	* It is proportional to $(\text{velocity})^n$, n varies from 1.72 to 2.0
* Independent of pressure	* Independent of pressure
* It is proportional to Area of surface in contact	* It is proportional to density of flow in fluid & Area of Surface in contact
* Independent of nature of surface in contact	* Independent of nature of Surface in Contact
* Greatly Effected by Temperature of variations on Flowing Fluid	* Slightly affected by temperature of variations on Flowing liquid.

Reynolds Experiment :—

the type of flow is determined by Reynolds number i.e

$$Re = \frac{\rho V * d}{\mu}.$$

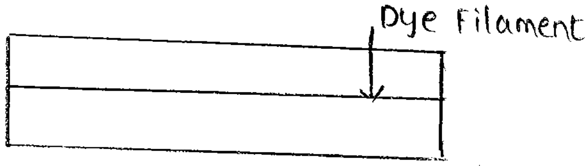
this was demonstrated by O. Reynold in 1883.



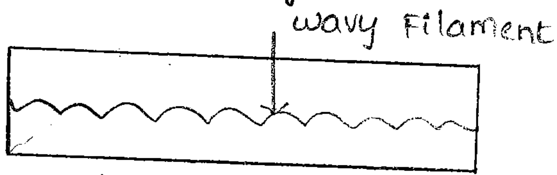
REYNOLD APPARATUS

the following observations were made by Reynold :

i) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye Filament was parallel to glass tube, which was the case of laminar flow as shown in below fig (a)



a) Laminar Flow



b) Transition

ii) With the Increase of velocity of flow, the dye Filament was no longer a straight line but it became a wavy nature as shown in above fig (b). This show no longer Laminar.

iii) With further Increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water as shown in Fig (c).

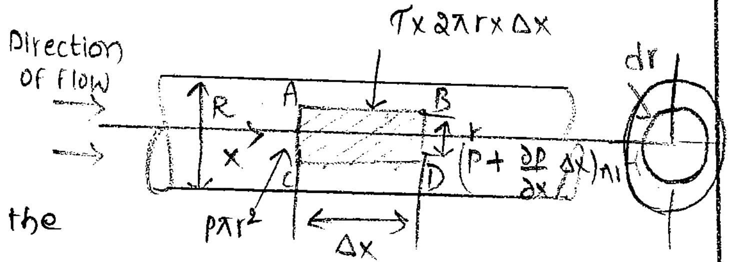
(c) Turbulent Flow

In case of Turbulent flow the mixing of dye - filament and water is intense and flow is irregular, Random & disorderly.

Laminar flow, Loss of pressure head is proportional to the velocity but in case of Turbulent flow, loss of Head is approximately Proportional to the square of velocity.

$\therefore$ Loss of Head $\boxed{h_f \propto V^n}$ $n \rightarrow 1.75$ to 2.0

SHEAR DISTRIBUTION IN LAMINAR FLOW :



Consider a Horizantal pipe of Radius 'R'. The viscous fluid is flowing from left to right in the pipe as shown in fig. consider a fluid element of radius 'r' sliding in a cylindrical element of radius $(r+dr)$. let the length of the fluid element be Δx. If 'p' is the Intensity on AB & $(p+\frac{\partial p}{\partial x}.\Delta x)$ on CD.

1. The pressure force, $p \times \pi r^2$ on face AB
2. The pressure force, $(p+\frac{\partial p}{\partial x}\Delta x)\pi r^2$ on face CD
3. shear force, $\tau \times 2\pi r \Delta x$ on surface of Fluid element.

As there is no accelration summation of all forces must be zero

$$p\pi r^2 - \left(p+\frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial p}{\partial x}.\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

$$-\frac{\partial p}{\partial x}.r - 2\tau = 0$$

$$\therefore \boxed{\tau = -\frac{\partial p}{\partial x}.\frac{r}{2}}$$

Velocity Distribution in Laminar flow :-

We know that $\tau = \mu \frac{du}{dy}$

y is measured from the pipe wall.

let $y = R - r$ i.e $dy = -dr$

$$\tau = \mu \frac{du}{-dr}$$

$$\tau = -\mu \frac{du}{dr}$$

As we know from shear stress, we derive $\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$

$$\cancel{+}\mu \frac{du}{dr} = \cancel{+}\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \cdot \frac{\partial p}{\partial x} \cdot r$$

By Integrating $\Rightarrow$ $u = \frac{1}{4\mu} \frac{\partial p}{\partial x} \cdot r^2 + c \rightarrow (1)$

By applying Boundary conditions, $r = R$, $u = 0$

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + c$$

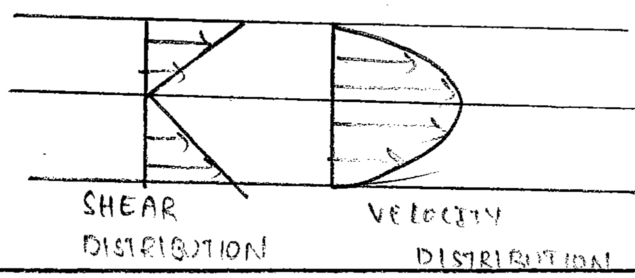
$$c = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

From (1) $\Rightarrow$ $u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$

$$u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} (R^2 - r^2)$$

u, $\frac{\partial p}{\partial x}$ & R are constant and velocity u varies with square of R

This equation implies a parabola



SHEAR STRESS DISTRIBUTION IN TURBULENT FLOW :

Reynolds in 1886 developed an expression for Turbulent shear Stress b/w two layers of a fluid at small distance apart

$$\tau = \rho u'v'$$

Where u' & v' = Fluctuating component of velocity in the direction of x & y due to Turbulence. As u' & v' changes, τ also changes. Hence to find shear stress time average on both sides of eq

$$\bar{\tau} = \overline{\rho u'v'}$$

This Turbulent stress is called "Reynold stress."

Turbulent Flow of Velocity Distribution :—

In pipes :—

In case of Turbulent flow, the total shear stress at any point is the sum of viscous shear stress & Turbulent shear stress. Viscous shear stress is neglible except near the Boundary.

$$\therefore \; u = \frac{u_*}{K} \log_e y + c \rightarrow (1)$$

$u_* \rightarrow$ Shear velocity

$u \rightarrow$ Velocity Distribution

$K \rightarrow$ Karman constant $(K = 0.4)$

By applying Boundary conditions,

$$y = R, \; u = u_{max}$$

$$u_{max} = \frac{u_*}{K} \log_e R + c$$

$$C = u_{max} - \frac{u_*}{K} \log_e R$$

From ① $\Rightarrow$ $u = \frac{u_*}{K} \log_e y + u_{max} - \frac{u_*}{K} \log_e R$

$$= u_{max} + \frac{u_*}{K} (\log_e y - \log_e R)$$

$$= u_{max} + \frac{u_*}{0.4} \log_e \left(\frac{y}{R}\right) \qquad [\because K = 4]$$

$$\therefore \boxed{u = u_{max} + 2.5\, u_* \log_e \left(\frac{y}{R}\right)}$$

this Equation is Applicable to both Rough & smooth pipes.

also written as, $u_{max} - u = -2.5\, u_* \log_e \left(\frac{y}{R}\right)$

$$u_{max} - u = 2.5\, u_* \log \left(\frac{R}{y}\right)$$

$\div$ by 'u_*' we get

$$\frac{u_{max} - u}{u_*} = 2.5 \log_e \left(\frac{R}{y}\right) \qquad \left[\because \log_e \left(\frac{R}{y}\right) = 2.3 \times \log_{10} \left(\frac{R}{y}\right)\right]$$

$$= 2.5 \times 2.3 \log_{10} \left(\frac{R}{y}\right)$$

$$\boxed{\frac{u_{max} - u}{u_*} = 5.75 \log_{10} \left(\frac{R}{y}\right)}$$

$u_{max} - u \rightarrow$ called Velocity Defect

Problem :-

1. A pipe-line carrying water has average height of irregularities projecting from the surface of the Boundary of the pipe as 0.15 mm. What type of Boundary it is? The shear stress developed is 4.9 N/m². The kinematic viscosity of water is 0.1 stokes

A:- Average Height of Irregularities $K = 0.15$ mm

$= 0.15 \times 10^{-3}$ m

shear stress $\tau_0 = 4.9$ N/m²

Kinematic viscosity $\nu = 0.01$ stokes $= 0.1 \times 10^{-4}\ m^2/s$

$\rho = 1000\ Kg/m^3$.

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07\ m/s$$

Roughness reynold number $= \frac{u_* k}{\nu} = \frac{0.07 \times 0.15 \times 10^{-3}}{0.1 \times 10^{-4}}$

$= 10.5$

$\frac{u_* k}{\nu}$ lies b/w 4 & 100.

Note :

Velocity Distribution for Turbulent flow in Rough pipes

$$\boxed{\frac{u}{u_*} = 5.75 \log_{10}\left(\frac{y}{k}\right) + 8.5}$$

Hagen poiseuille Formula :—

Already we proved in velocity Distribution of Laminar flow

$$u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2]$$

When $r=0$, $u = u_{max}$ then $u_{max} = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 \rightarrow (1)$

The average velocity $\bar{u}$ is obtained by dividing discharge of Fluid across the section by the Area of the pipe. The Discharge is obtained by considering flow through a circular ring element of radius r & thickness dr

dQ = velocity at a radius r * Area of ring element

$= u \times 2\pi r\, dr$

$$= -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \times 2\pi r\, dr$$

$$Q = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2-r^2)\times 2\pi r\,dr$$

$$= -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)\times 2\pi\int_0^R (R^2-r^2)r\,dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\times 2\pi\int_0^R (R^2r-r^3)\,dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\times 2\pi\left[\frac{R^2r^2}{2}-\frac{r^4}{4}\right]_0^R$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)\times 2\pi\left[\frac{R^4}{2}-\frac{R^4}{4}\right]$$

$$Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$$

$\therefore$ Average velocity $\bar{u} = \dfrac{Q}{\text{Area}}$

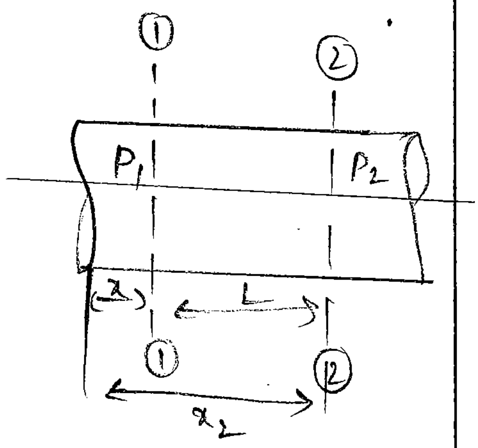
$$\bar{u} = \frac{\frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4}{\pi R^2}$$

$$\therefore\ \bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \longrightarrow ②$$

$$\frac{①}{②} \Rightarrow \frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{-\frac{1}{8\mu}\frac{\partial p}{\partial x}R^2}$$

$$\therefore\ \boxed{\frac{u_{max}}{\bar{u}} = 2.}$$

From ② $\Rightarrow$ $-\dfrac{\partial p}{\partial x} = \dfrac{8\mu\bar{u}}{R^2}$



Integrating above w.r.t x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$-[P_1-P_2] = \frac{8\mu\bar{u}}{R^2}[x_1-x_2]$$

$$P_1 - P_2 = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1] \quad \left[\text{from dig } x_2 - x_1 = L\right]$$

$$P_1 - P_2 = \frac{8\mu\bar{u}}{R^2}(L)$$

$$= \frac{8\mu\bar{u}}{\left(\frac{D}{2}\right)^2}(L)$$

$$\therefore P_1 - P_2 = \frac{32\mu\bar{u}}{D^2} \cdot L \qquad P_1 - P_2 \rightarrow \text{Drop pressure}$$

$$\text{Loss of Head} = \frac{P_1 - P_2}{\rho g}$$

$$\therefore \boxed{\frac{P_1 - P_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}}$$

∴ this eq is called Hagen poiseuille formule

1. A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a Horizantal circular pipe of diameter 100 mm and of length 10 m. calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank of 30 sec ?

A)

$$\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$$

Relative density = 0.9

$$\rho_o = 0.9 \times 1000 = 900 \text{ kg/m}^3$$

Dia of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

$$L = 10 \text{ m}$$

Mass of oil collected, $M = 100 \text{ kg}$

$$t = 30 \text{ sec}$$

$$P_1 - P_2 = \frac{32\mu\bar{u}L}{D^2}$$

$\bar{u}$ = Avg velocity = $\frac{Q}{\text{Area}}$

mass = $\frac{100}{30}$ ~~kg~~ /s

$= \rho_0 \times Q$

$\frac{100}{30} = 900 \times Q$

$\therefore Q = 0.0037 \ m^3/s$

$\bar{u} = \frac{Q}{A} = \frac{0.0037}{\frac{\pi}{4}D^2} = 0.471 \ m/s \quad (\because D = 0.1)$

For viscous flow $Re < 2000$

$\therefore Re^* = \frac{\rho V D}{\mu} \qquad V = \bar{u}$

$Re = \frac{900 \times 0.471 \times 0.1}{0.097}$

$= 436.91 \ (< 2000)$

$\therefore P_1 - P_2 = \frac{32 \mu \bar{u} L}{D^2}$

$= \frac{32 \times 0.097 \times 0.471 \times 10}{(0.1)^2}$

$P_1 - P_2 = 0.1462 \ N/cm^2$

Loss of Head Due to Friction in Viscous Flow in relation With Reynolds number :-

From Hagen poiseullie formula, $h_f = \frac{32 \mu \bar{u} L}{\rho g D^2} \rightarrow (1)$

Loss of Head due to Friction is given by

$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{D \times 2g} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times 2g} \rightarrow (2) \quad \because V = \bar{u}$

f → coefficient of Friction b/w Pipe & fluid

(1) = (2),

$\frac{32 \mu \bar{u} L}{\rho g D^2} = \frac{4 \cdot f \cdot L \cdot \bar{u}^2}{D \times 2g}$

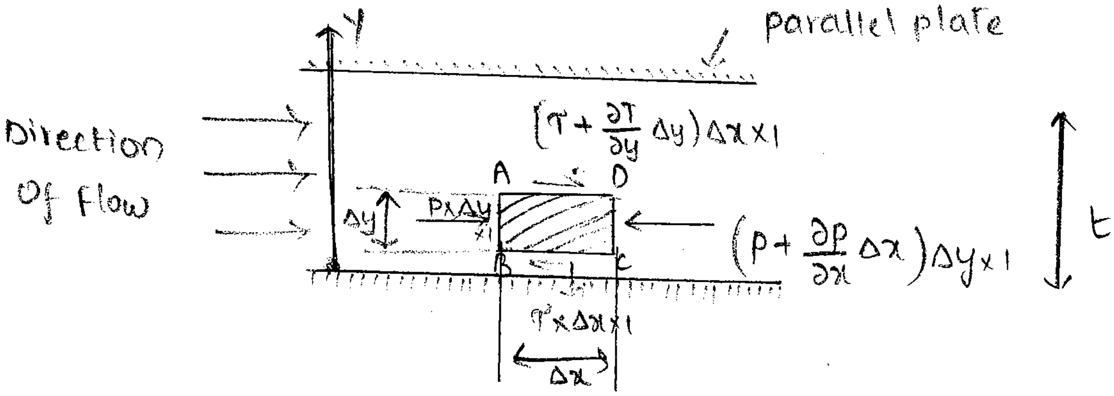
$$f = \frac{16\mu}{\rho \cdot D \cdot \bar{u}}$$

$$f = 16 \times \frac{\mu}{\rho V D} \qquad (\because \bar{u} = V)$$

$$= 16 \times \frac{1}{\left(\frac{\rho V D}{\mu}\right)}$$

$$\boxed{f = \frac{16}{Re}} \qquad \left(\because Re = \frac{\rho V D}{\mu}\right)$$

Flow of viscous flow between Two parallel plates :-



Consider a fluid element of length 'Δx' & thickness 'Δy' at a distance 'y' from lower fixed plate. If P is Intensity of Pressure on AB & $(P + \frac{\partial P}{\partial x} \Delta x)$ on CD. let τ is shear stress on BC and $(\tau + \frac{\partial P}{\partial y} \cdot \Delta y)$. on AD

1. Pressure force , $P \times \Delta y \times 1$ on AB
2. pressure force $(P + \frac{\partial P}{\partial x} \Delta x) \Delta y \times 1$ on CD
3. shear force $\tau \times \Delta x \times 1$ on BC
4. shear force $(\tau + \frac{\partial P}{\partial y} \Delta y) \Delta x \times 1$ on AD

there is no accerlation and hence resultant direction flow is zero.

$$P \times \Delta y \times 1 - \left(P + \frac{\partial P}{\partial x} \Delta x\right) \Delta y \times 1 - \tau \times \Delta x \times 1 + \left(\tau + \frac{\partial P}{\partial y} \Delta y\right) \Delta x \times 1 = 0$$

$$-\frac{\partial P}{\partial x} \Delta x \Delta y + \frac{\partial \tau}{\partial x} \cdot \Delta y \Delta x = 0$$

$\div$ by $\Delta x \Delta y$, $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$.

i) Velocity Distribution:

From $\tau = \mu \frac{du}{dy}$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu \frac{du}{dy}\right) \quad \text{From} \left(\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}\right)$$

$$= \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{dy^2} = \frac{1}{\mu} \frac{\partial p}{\partial x}$$

Integrating above eq w.r.t 'y'

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial p}{\partial x} y + c_1$$

Again, $u = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{y^2}{2} + c_1 y + c_2 \rightarrow (1)$ $\left(\because \frac{\partial p}{\partial x} = \text{constant}\right)$

By applying Boundary conditions,

At $y=0, u=0 \Rightarrow 0 = \frac{1}{\mu} \frac{\partial p}{\partial x} \times 0 + c_1 \times 0 + c_2$

$$c_2 = 0$$

At $y=t, u=0 \Rightarrow 0 = \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{t^2}{2} + c_1 \times t + 0 \quad (\because c_2 = 0)$

$$\therefore c_1 = -\frac{1}{2\mu} \frac{\partial p}{\partial x} t$$

From (1) $\Rightarrow u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + y\left(-\frac{1}{2\mu} \frac{\partial p}{\partial x} t\right)$

$$\boxed{\therefore u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} \left[ty - y^2\right]}$$

ii) Ratio of maximum velocity to Average velocity:

When $y = \frac{t}{2}$, $u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} \left[t \times \frac{t}{2} - \left(\frac{t}{2}\right)^2\right]$

$$\therefore u_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}t^2 \rightarrow (2)$$

we know discharge = velocity * Area

dQ = velocity at distance y * Area of strip

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty-y^2] * dy * 1$$

$$Q = \int_0^t dQ = \int_0^t -\frac{1}{2\mu}\frac{\partial p}{\partial x}[ty-y^2]\,dy$$

$$= -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[\frac{ty^2}{2}-\frac{y^3}{3}\right]_0^t$$

$$= -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3$$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{-\frac{1}{12\mu}\frac{\partial p}{\partial x}t^3}{t \times 1} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}t^2 \rightarrow (3)$$

$$\frac{(2)}{(3)} \Rightarrow \frac{u_{max}}{\bar{u}} = \frac{-\frac{1}{8\mu}\frac{\partial p}{\partial x}\cdot t^2}{-\frac{1}{12\mu}\frac{\partial p}{\partial x}\cdot t^2}$$

$$\boxed{\frac{u_{max}}{\bar{u}} = \frac{3}{2}}$$

Drop of pressure head for given length :-

$$(3) \Rightarrow \bar{u} = -\frac{1}{12\mu}\frac{\partial p}{\partial x}\cdot t^2$$

$$\frac{\partial p}{\partial x} = -\frac{12\mu\bar{u}}{t^2}$$

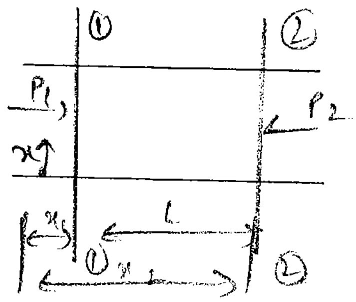
Integrating w.r.t 'x'

$$\int_2^1 dp = \int_2^1 -\frac{12\mu\bar{u}}{t^2}\,dx$$

$$P_1 - P_2 = -\frac{12\mu\bar{u}}{t^2}[x_1 - x_2]$$

$$P_1 - P_2 = \frac{12\mu\bar{u}}{t^2}[x_2 - x_1]$$

$$P_1 - P_2 = \frac{12\mu \bar{u} L}{t^2} \quad \because x_2 - x_1 = L$$



$$\boxed{h_f = \frac{P_1 - P_2}{\rho g} = \frac{12\mu \bar{u} L}{\rho g t^2}}$$

iv) Shear stress Distribution :-

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{\partial}{\partial y}\left[-\frac{1}{2\mu} \frac{\partial p}{\partial x} (ty - y^2)\right]$$

$$\tau = -\frac{1}{2}[t - 2y] \frac{\partial p}{\partial x}$$

By Apply Boundary conditions , $y = 0$

$$\boxed{\therefore \tau = -\frac{1}{2} \frac{\partial p}{\partial x} \cdot t}$$

1. Calculate i) pressure gradient along flow ii) the average velocity iii) the discharge for oil of viscosity 0.02 Ns/m² flowing b/w two stationary parallel plates 1 m wide maintained 10mm apart. The velocity midway b/w plates is 2 m/s ?

A: viscosity $\mu = 0.2$ N·s/m², $b = 1$m , $t = 10$ mm $= 0.1$m

$u_{max} = 2$ m/s

i) pressure gradient

$$u_{max} = -\frac{1}{8\mu} \frac{\partial p}{\partial x} \cdot t^2$$

$$2 = -\frac{1}{8 \times 0.2} \frac{\partial p}{\partial x} (0.1)^2$$

$$\frac{\partial p}{\partial x} = -3200 \text{ N/m}^2$$

ii)

$$\frac{u_{max}}{\bar{u}} = \frac{3}{2}$$

$$\therefore \bar{u} = \frac{2}{3} \times u_{max} = \frac{2}{3} \times 2$$

$$\therefore \bar{u} = 1.33 \text{ m/s}$$

iii) $Q = \bar{u} \times \text{Area} = b \times t \times \bar{u} = 1 \times 0.1 \times 1.33$

$Q = 0.0133$ m³/s

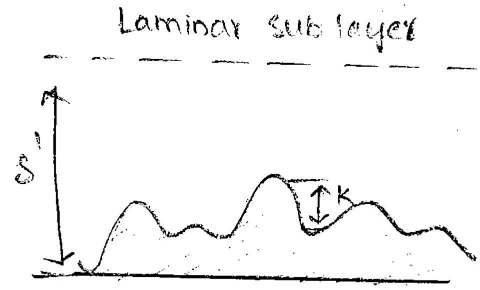
Hydrodynamically Smooth & Rough pipes :—

let K is the average height of irregularities projecting from the surface of a Boundary

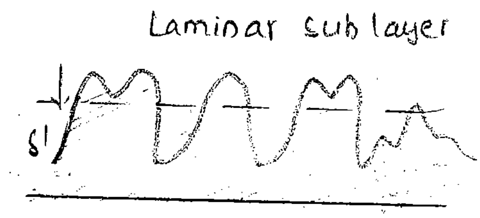
If value of K is large then Boundary is called Rough Boundary

If value of K is small than Boundary is called smooth Boundary

classification is based on Boundary characteristics



a) Smooth Boundary



b) Rough Boundary.

Viscous shear force predominates while the shear stress due to turbulence is negligible. This portion is known as Laminar sub layer. When shear stress due to turbulence are large as compared to viscous stress is known as Turbulent zone.

The Eddies present in turbulent flow try to penetrate the laminar sub layer and reach surface of Boundary but due to great thickness of laminar sub layer the eddies are unable to reach the surface irregularities. This Boundary is called "Hydrodynamically smooth Boundary." In this Reynolds number is much less as shown in fig (a)

If the Reynolds number of flow is Increased then thickness of laminar sub layer will decrease. The thickness of laminar sub layer becomes much smaller than average height 'K' boundary acts as rough Boundary and irregularities of surface are become above the laminar sublayer and eddies present in Turbulent zone will come in contact with surface & lot of energy will be lost. this is called Hydrodynamically Rough Boundary

→ If $\frac{K}{\delta'}$ is less than 0.25 → smooth

→ If $\frac{K}{\delta'}$ is greater than 6.0 → Rough

→ $0.25 < \frac{K}{\delta'} < 6$ → Transition

* Roughness Reynolds Number = $\frac{u_* K}{\nu}$

→ If $\frac{u_* K}{\nu} < 4$ → smooth

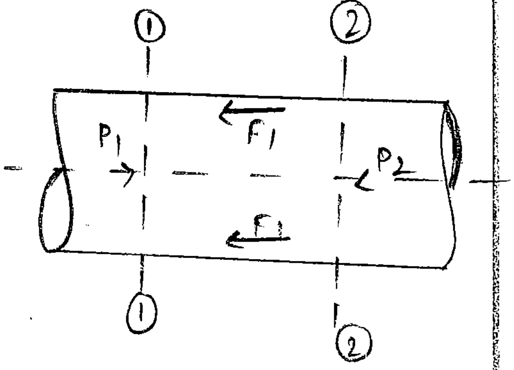
→ $\frac{u_* K}{\nu}$ lies b/w 4 & 100 → Transition stage

→ $\frac{u_* K}{\nu} > 100$ → Rough

Mr. P. Siva Sankar
M. Tech (SE)
Asst. Prof.

Darcy - Weisbach Equation :—

Consider uniform Horizontal pipe, having steady flow as shown in fig. let 1-1 & 2-2 are two sections of pipe



let p_1 = pressure Intensity at section 1-1

V_1 = velocity of flow at section 1-1

L = length of pipe between sections 1-1 & 2-2

d = diameter of pipe

f' = Frictional resistance per unit wetted Area per unit velocity

P_2, V_2 = pressure Intensity & velocity of flow at 2-2

Applying Bernoullis Eq's b/w 1-1 & 2-2,

Total head at 1-1 = Total head at 2-2 + Loss of Head due to Friction b/w 1-1 & 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is Horizontal

$V_1 = V_2$ as dia of pipe is same at both sections

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \quad (\text{or}) \quad h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \rightarrow (1)$$

Intensity of pressure will be reduced in the direction of flow by Frictional resistance

$$F_1 = f' \times \pi d L \times V^2$$

Frictional Resistance = Frictional resistance per unit wetted Area per unit velocity * wetted Area * (velocity)^2

∴ wetted Area = $\pi d \times L$

Velocity $V = V_1 = V_2$

πd = perimeter = p

$$F_1 = p \times L \times V^2 \rightarrow (2)$$

The Forces acting on the fluid between sections 1-1 & 2-2 are:

1. Pressure Force at section 1-1 = $P_1 \times A$, $A \rightarrow$ Area of pipe
2. Pressure Force at section 2-2 = $P_2 \times A$
3. Frictional force F_1 as

Resolving all forces in Horizontal direction,

$$P_1A - P_2A - F_1 = 0$$

$$(P_1 - P_2)A = F_1 = f' \times P \times L \times V^2$$

$$P_1 - P_2 = \frac{f' \times P \times L \times V^2}{A} \qquad \because F_1 = f'PLV^2$$

From ① $\Rightarrow$ $P_1 - P_2 = \rho g h_f$

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \rightarrow ③$$

In Eq ③, $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4}d^2} = \frac{4}{d}$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2$$

$$\therefore h_f = \frac{f'}{\rho g} \cdot \frac{4LV^2}{d} \rightarrow ④$$

$\frac{f'}{\rho} = \frac{f}{2}$, $f \rightarrow$ coefficient of friction

④ as, $h_f = \frac{f}{2g} \cdot \frac{4LV^2}{d}$

$$\boxed{\therefore h_f = \frac{4fLV^2}{2gd}}$$

- This Eq is called Darcy-weisbach Equation

Sometimes Eq is written as

$$\boxed{h_f = \frac{f \cdot L \cdot V^2}{2g \times d}}$$

Chezys formula for loss of Head due to Friction in pipes :—

We know loss of head $h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$

$h_f \rightarrow$ loss of head due to friction, P = Wetted perimeter of pipe

$A \rightarrow$ Area of cross section of pipe, L = length of pipe

$V \rightarrow$ Mean velocity of flow.

the ratio of $\frac{A}{P} = \frac{\text{Area of flow}}{\text{Wetted perimeter}}$ is called Hydraulic mean depth (or) Hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

$$\frac{A}{P} = \frac{m}{1} \quad \text{(or)} \quad \frac{P}{A} = \frac{1}{m}$$

$$h_f = \frac{f'}{\rho g} \times \frac{1}{m} \times L \times V^2 \Rightarrow V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L}$$

$$V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} \Rightarrow V = \sqrt{\frac{\rho g}{f'}}\sqrt{m.\frac{h_f}{L}} \rightarrow (1)$$

let $\sqrt{\frac{\rho g}{f'}} = C$, Where $C \rightarrow$ chezys constant

$\frac{h_f}{L} = i$, $i \rightarrow$ loss of Head per unit length of pipe

By substituting in (1), $\boxed{V = C\sqrt{mi}}$

this Eq is called chezys formula.

m is always equal to $\frac{d}{4}$

1. Find the head lost due to friction in a pipe of diameter 300 mm & length 50 m through which water is flowing at a velocity of 3 m/s using i) Darcy Formula ii) chezys formula, $C = 60$
Take ν for water = 0.01 stoke

A:- Dia of pipe d = 300 mm = 0.30 m

Length of pipe L = 50 m

Velocity = 3 m/s , $C = 60$,

Kinematic viscosity $\nu = 0.01 \times 10^{-4}$ m²/s

i) Darcy formula $h_f = \dfrac{4 \cdot f \cdot L \cdot v^2}{d \times 2g}$

f = coeffcient of friction is a friction of Reynolds number

$$Re = \frac{V \times d}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$\therefore f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

Head lost, $h_f = \dfrac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2 \times 9.81}$

$$h_f = 0.7828 \text{ m}$$

ii) chezys formula $V = C\sqrt{mi}$

$$C = 60, \quad m = \frac{d}{4} = \frac{0.30}{4} = 0.075 \text{ m}$$

$$3 = 60\sqrt{0.075 \times i} \Rightarrow i = 0.0333$$

$$i = \frac{h_f}{L} \Rightarrow \frac{h_f}{50} = 0.0333$$

$$h_f = 1.665 \text{ m}$$

Note for Darcy Eq :

f = Coebbcient of friction which is a function of Reynolds number

$$f = \frac{16}{Re} \quad \text{for} \quad Re < 2000$$

$$f = \frac{0.079}{(Re)^{1/4}} \quad Re \text{ varying } 4000 \text{ to } 10^6$$

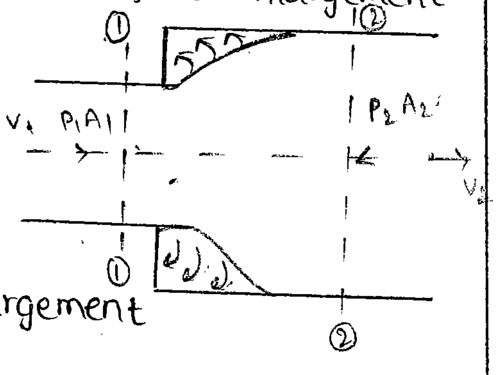
Minor Energy Losses :—

The Loss of Head (or) energy due to friction in a pipe is known as Major Loss while the loss of Energy due to change of velocity of following fluid in magnitude (or) direction is called Minor Loss of Energy. these includes following cases :

1. Loss of Head due to Sudden Enlargement
2. Loss of Head due to Sudden contraction
3. Loss of Head at Entrance of a pipe
4. Loss of Head at Exit of a pipe
5. Loss of Head due to an obstruction in a pipe
6. Loss of Head due to bend in the pipe
7. Loss of Head in various pipe Fittings

Loss of Head due to Sudden Enlargement :—

Consider a liquid flowing through a pipe which has sudden Enlargement. consider two sections before and after enlargement



P_1, P_2 = Pressure intensities at ①-① & 2-2

V_1, V_2 = Velocity of flows at 1-1 & 2-2

A_1, A_2 = Area of pipes at 1-1 & 2-2

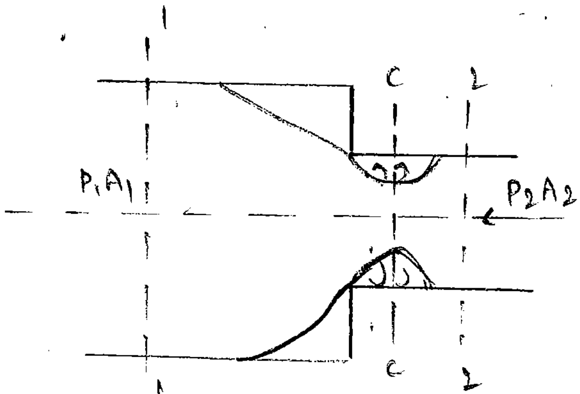
h_e = Loss of Head due to sudden enlargement

$$\boxed{h_e = \frac{(V_1-V_2)^2}{2g}}$$

Loss of Head due to Sudden contraction :—

consider a liquid flowing in a pipe which has a sudden contraction in area as shown. consider two sections before and after contraction As the liquid flows from larger pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section c-c.

This section c-c is known as Vena-contracta. After section c-c sudden enlargement is takes place. The Loss of Head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe



A_c = Area of flow at c-c

V_c = Velocity of flow at c-c

A_2 = Area of flow at 2-2

V_2 = Velocity of flow at 2-2

h_c = Loss of Head due to sudden contraction

$$\boxed{h_c = 0.5\frac{V_2^2}{2g}}$$

~~Velocity~~ Loss of Head at Entrance of Pipe :-

This is loss of Energy which occurs when a liquid enters a Pipe which is connected to a large tank (or) reservoir. This loss is similar to the loss of Head due to sudden contraction. This loss depends on the form of Entrance. For a sharp edge entrance, this loss is slightly more than a rounded (or) bell mouthed Entrance. In practice the value of loss of Head at Entranance of a pipe with sharp cornered entrance is taken $= 0.5\frac{V^2}{2g}$,

$$\boxed{h_i = 0.5\frac{V^2}{2g}}$$

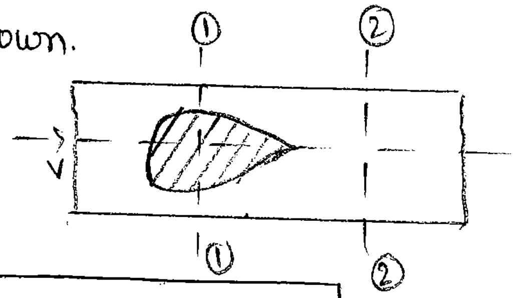
Loss Of Head at Exit of a Pipe :-

This is loss of Head due to the Velocity of liquid at outlet of Pipe which is dissipated in the form of free jet or it is lost in the tank or reservoir. This loss is equal to $\frac{V^2}{2g}$, It is denoted by h_o

$$\boxed{h_o = \frac{V^2}{2g}}$$

Loss of Head due to obstruction in a pipe :-

Whenever there is an obstruction in a pipe, the loss of energy takes place due to the reduction of the Area of cross section of the pipe at place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown.



a = max area of obstruction

A = Area of pipe

V = velocity of liquid in pipe

Head loss due to obstruction = $\boxed{\frac{V^2}{2g}\left(\frac{A}{C_c(A-a)}-1\right)^2}$

C_c = coefficient of contraction

Loss of Head due to Bend in pipe :-

When there is any bend in a pipe, the velocity of flow changes due to which the separation of flow from the boundary and also formation of eddies takes place. Thus energy is lost

$$h_b = \frac{KV^2}{2g}$$

$h_b \rightarrow$ loss of head due to bend

$V \rightarrow$ velocity of flow

$K \rightarrow$ coefficient of Bend

K Depends on

→ Angle of Bend, Radius of curvature & Diameter of pipe

Loss of Head in various pipe fittings :-

The loss of Head in the various pipe fittings such as valves, couplings etc... is expressed as

$$\frac{KV^2}{2g},$$

$K \rightarrow$ coefficient of pipe fitting

$V \rightarrow$ velocity of flow

1. The difference in water surface levels in two tanks which are connected by three pipes in series of length 300 m, 170 m & 210 m and of diameter 300 mm, 200 mm & 400 mm respectively is 12 m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 & 0.0048 respectively, considering

i) Minor losses also & ii) Neglecting Minor Losses

A:- Given, Difference $H = 12m$

$L_1 = 300m \quad L_2 = 170\,m, \quad L_3 = 210\,m$

$D_1 = 300\,mm = 0.3\,m \quad D_2 = 200\,mm = 0.2\,m \quad D_3 = 400\,mm = 0.4\,m$

$f_1 = 0.005 \quad f_2 = 0.0052 \quad f_3 = 0.0048$

i) considering Minor losses

V_1, V_2 & V_3 are velocities of pipes 1, 2 & 3

From continuity eq, $A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} \times V_1$$

$$\therefore V_2 = 2.225V_1$$

$$V_3 = \frac{A_2V_1}{A_3} = \frac{d_1^2}{d_3^2} \times V_1$$

$$\therefore V_3 = 0.5625V_1$$

We have Eq for minor Losses when they are considered

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g \times d_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2g \times d_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 & V_3 :

$$12 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{2 \times g \times 0.3} + \frac{0.5 \times (2.225V_1^2)^2}{2g} +$$

P.T.O

$$+ \frac{4\times0.0052\times A_0\times 2.225(v_1^2)}{0.2\times2g} + \frac{(2.25v_1-0.562v_1)^2}{2g} + \frac{4\times0.0048\times210\times(0.5625v_1)^2}{0.4\times2g}$$

$$+ \frac{(0.5625v_1)^2}{2g}$$

$$12 = \frac{v_1^2}{2g}[118.887) \Rightarrow v_1 = 1.407 \text{ m/s}$$

Rate of flow Q = Area * velocity

$$= \frac{\pi}{4}d^2 * v_1$$

$$= (0.3)^2\frac{\pi}{4} * 1.407$$

$\therefore Q = 99.45$ liter/sec

ii) Neglecting Minor Losses :

$$H = \frac{4f_1L_1v_1^2}{2gd_1} + \frac{4f_2L_2v_2^2}{2gd_2} + \frac{4f_3L_3v_3^2}{2gd_3}$$

$$12 = \frac{v_1^2}{2g}\left(\frac{4\times0.005\times300}{0.3} + \frac{4\times0.0052\times90\times(2.25)^2}{0.2} + \frac{4\times0.0048\times210\times(0.5625)^2}{0.4}\right)$$

$$12 = \frac{v_1^2}{2g}[112.694] \Rightarrow v_1 = 1.445 \text{ m/sec}$$

Discharge $Q = v_1 * A_1$

$$= \frac{\pi}{4}(0.3)^2 * 1.445$$

$\therefore Q = 102.1$ liters/sec

Flow through Pipes in series (or) Flow through compound pipes

Pipes in series (or) compound pipes are defined as the pipes of different lengths & different diameters connected end to end to form a pipe line.

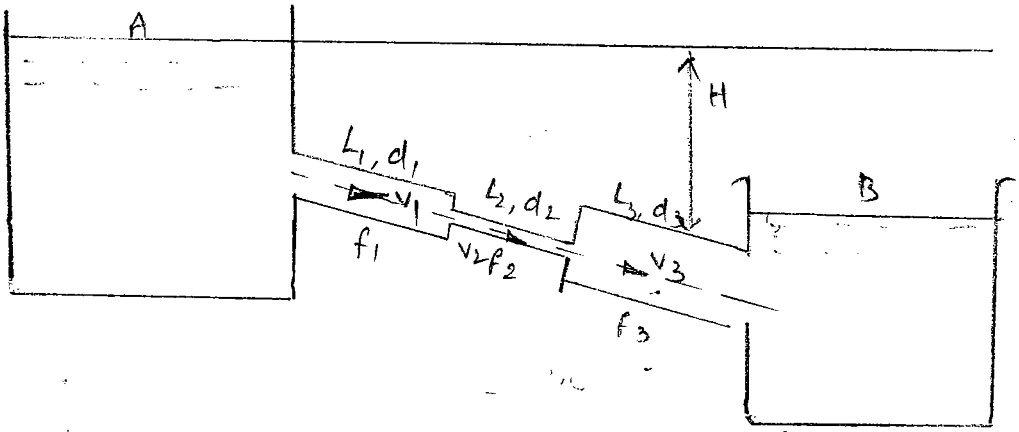
let l_1, l_2, l_3 = length of pipes 1, 2 & 3 respectively

d_1, d_2, d_3 = diameter of pipes 1, 2 & 3 respectively

v_1, v_2, v_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = coeffcients of friction for pipes 1, 2, 3

H = difference of water level in two tanks.



The Discharge passing through each pipe is same

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

the difference in liquid surface levels is equal to the sum of Total head loss in the pipes

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2g \times d_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g \times d_3} + \frac{V_3^2}{2g} \rightarrow (1)$$

If minor Losses are neglected then Eq becomes

$$H = \frac{4f_1L_1V_1^2}{2g \times d_1} + \frac{4f_2L_2V_2^2}{2g \times d_2} + \frac{4f_3L_3V_3^2}{2g \times d_3}$$

If coefficient of friction is same for pipes

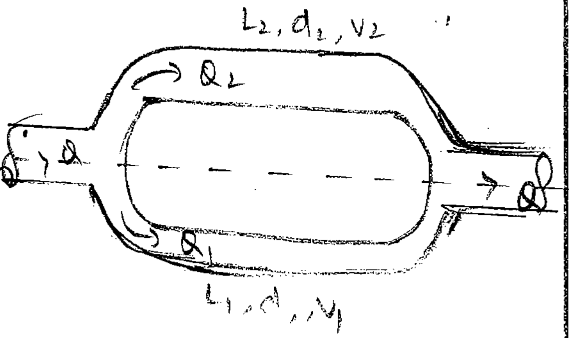
$$f_1 = f_2 = f_3 = f$$

$$H = \frac{4fL_1V_1^2}{2g \times d_1} + \frac{4fL_2V_2^2}{2g \times d_2} + \frac{4fL_3V_3^2}{2g \times d_3}$$

$$\boxed{H = \frac{4f}{2g}\left(\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right)}$$

Flow through parallel plates :—

Consider a main pipe which divides into two (or) more branches as shown in fig and again Join together downstream to form a single pipe then the Branch pipes are said to be connected in parallel. the discharge through the main is increased by connecting Pipes in parallel



The rate of flow in main pipe is equal to the sum of rate of flow through branch Pipes. We have

$$Q = Q_1 + Q_2$$

Loss of Head for branch pipe 1 = Loss of Head for branch pipe 2

$$\frac{4f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\boxed{\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}} \quad \because f_1 = f_2$$

Energy :

* The ability to do work
* the ability to exert force on an object to move it.

Pressure Energy :

→ It is a force that produce the flow in a pipe line
→ It is expressed in terms of Head i.e in metres of liquid
→ It is equal to the pressure (p) divided by specific wt of liquid
→ pressure head = $\frac{P}{W}$

Potential Energy :

* It is energy stored by liquid due to its elevated position
* It is expressed in terms of Head i.e in meters
* It is equal to the Height of liquid from datum

* Potential Energy Head = z meters

Kinetic Energy :—

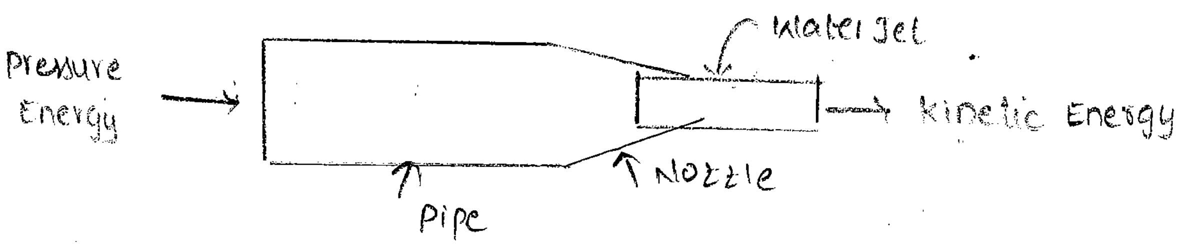
→ A moving liquid possess kinetic Energies

→ It is the energy possessed by the liquid due to its motion

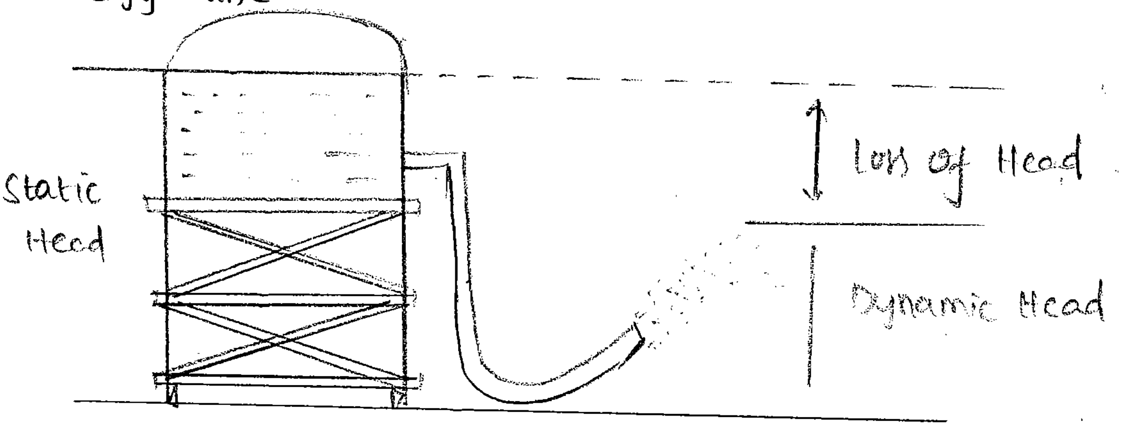
→ It is expressed in terms of head of liquid i.e $\frac{v^2}{2g}$ in meters

Kinetic Energy Head = $\frac{v^2}{2g}$ meters

During the fluid flow the change of energy from one form to the other takes place.



this can be shown graphically by using Hydraulic gradient line and Total Energy line

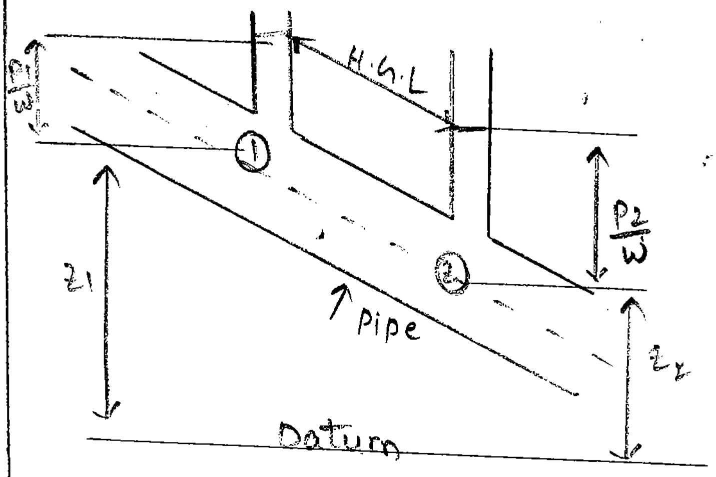


Hydraulic Gradient line :—

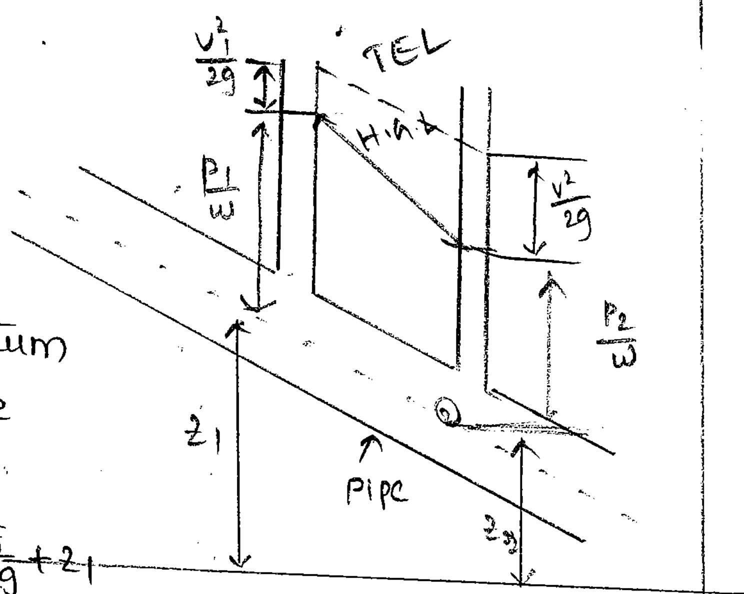
It is the line Joining the sum of pressure Heads & datum Heads at different sections of the pipe.

Piezometric Head at section ① = $\frac{P_1}{w} + z_1$

Piezometric Head at section ② = $\frac{P_2}{w} + z_2$



HGL Joins $\frac{P_1}{w} + z_1$ with $\frac{P_2}{w} + z_2$ as shown in fig



Total Energy line :—

It is the line Joining the sum of Pressure Head, velocity Head & datum Head at different sections of the pipe.

Total Energy at section ① $= \frac{P_1}{w} + \frac{v_1^2}{2g} + z_1$

Total Energy at section ② $= \frac{P_2}{w} + \frac{v_2^2}{2g} + z_2$

TEL Joins $\frac{P_1}{w} + \frac{v_1^2}{2g} + z_1$ with $\frac{P_2}{w} + \frac{v_2^2}{2g} + z_2$

Pipe Network :—

A pipe network is an incorrected system of pipes forming several loops (or) circuits. The ~~pipe~~ examples of such networks of pipes are the municipal water distribution systems in cities and labaratory supply system. In such system, it is required to determine the distribution of flow through various pipes of network

(i) The flow into each Junction must be equal to the flow out of the Junction. This is due to continuity Equation

ii) The algebraic sum of Head losses round each loop must be zero. this means that in each loop, the loss of head due to flow in clockwise direction must be equal to the loss of head due to flow in anticlockwise direction.

iii) the head loss in each pipe is expressed as $h_f = rQ^n$. The value of r depends on the length, Diameter & co-efficient of Friction of pipe.

Value of n for Turbulent flow is "2"

$$h_f = \frac{4 \times f \times L \times v^2}{D \times 2g} \qquad \left(v = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}\right)$$

$$= \frac{4fL \times \left(\frac{Q}{A}\right)^2}{D \times 2g}$$

$$= \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4}D^2\right)^2}$$

$$= \frac{4fLQ^2}{2g \times \left(\frac{\pi}{4}\right)^2 D^5}$$

$$\boxed{h_f = rQ^2} \qquad \left(\because r = \frac{4fL}{2g\left(\frac{\pi}{4}\right)^2 D^2}\right)$$

Hardy cross Method :—

The procedure for Hardy cross method is as follows :

1. In this method a trail Distribution of discharges is made arbitary but in such a way that continuity eq is satisfied at each Junction.

2. With the Assumed values of Q, the head loss in each pipe is calculated according to the equation $h_f = rQ^2$

3. Now consider any loop. The algebraic sum of head losses round each loop must be zero. This means that in each loop the loss of head due to flow in clockwise direction must be

equal to loss of Head due to flow in anticlockwise direction.

4. Now calculate the net head loss around each loop considering the head loss to be positive in clockwise flow and to be negative in anticlockwise flow.

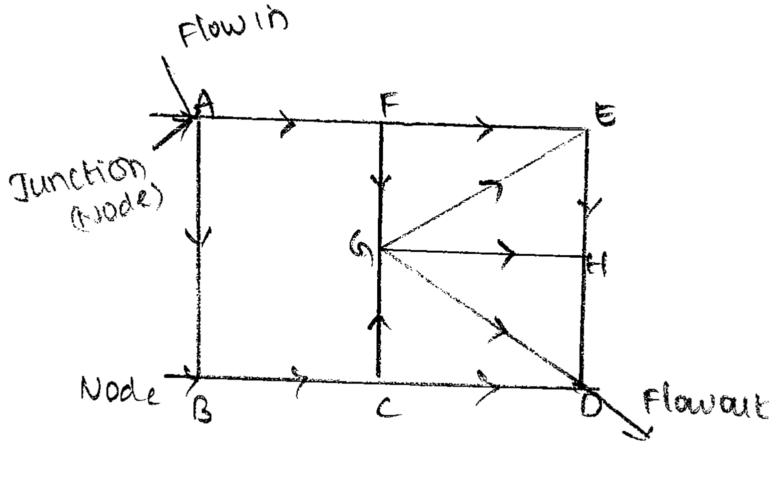
If the net head loss due to assumed Q values in the loop is zero, then the assumed values of Q in that loop is correct. But if the net head loss due to assumed values of Q is not zero then the Assumed values of Q are corrected by introducing a correction ΔQ for the flows

$$\text{Correction factor } \Delta Q = \frac{-\Sigma r Q_0^n}{\Sigma r \cdot n \cdot Q_0^{n-1}}$$

5. If the value of ΔQ comes out to be positive, then it should be added to the flows in clockwise direction and subtracted from the flows in anticlockwise direction.

6. Some pipes may be common to two circuits, then the two corrections are applied to these pipes.

7. After the corrections have been applied to each pipe in a loop and to all loops, a second trial calculation is made for all loops. The procedure is repeated till ΔQ becomes neglible.



Loops : ABCGFA, FEGF, GEHG, GHDG & GCDG

UNIT-IV

Mr. P. Siva Sankar, M.Tech (Struc
Asst. Prof. of C.E. Department

Lecture Handout

FLOWS

Objectives:

At the end of this topic, you will be able to:

→ Describe Reynold's experiment
→ Differentiate the types of flow
→ Explain flow between parallel plates
→ Detail on flow through long plates
→ Describe flow through inclined plates
→ Solve problems based on the aforesaid concepts.

Outcomes:

By the end of this topic, you will be able to.

→ Understand Reynold's experiment
→ Know about the types of flow
→ Understand the flow between parallel plates
→ know about the flow through long plates and inclined plates.
→ Illustrate problems based on the aforesaid concepts.

Introduction:

→ A pipe is a closed conduit which is used for carrying fluid under pressure

→ Pipes are commonly circular in section.

→ As the pipes carry fluids under pressure, the pipes alway run full.

→ The fluid flowing in a pipe is always subjected to resistanc due to shear forces.

→ It occurs between fluid particles and the boundary walls of the pipe and between the fluid particles themselves resulting from the viscosity of the fluid.

→ The resistance to the flow of fluid is general known as frictional resistance.

→ Since Certain amount of energy possessed by the flowing fluid will be consumed in overcoming this resistance to the flow, there will always be some loss of energy in the direction of flow.

→ Loss of energy depends on the type of flow. The flow of fluid in a pipe may be either laminar or turbulent

→ The existence of the two types of flow, viz., Laminar and turbulent, was first demonstrated by Osborne Reynold's in 1883, with the help of a simple experiment discussed.

Reynold's Experiment :

→ The setup shown in figure consists of the following

* Constant head tank filled with water
* small tank containing dye
* Glass tube with a bell mouthed entrance
* Regulating Valve

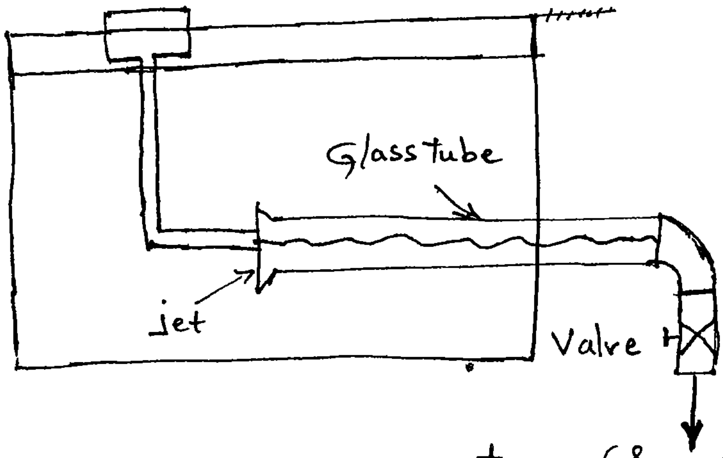


Fig. Reynold's apparatus for demonstrating the types of flow.

→ The water was made to flow from the tank through the glass tube into atmosphere and the velocity of flow was varied by adjusting the regulating valve.

→ A liquid dye having the same specific weight as that of water, was introduced into the flow at the bell-mouth through a small tube.

Conclusions

→ From the experiments it was seen that when the velocity of flow was low.

→ The dye remained in the form of a straight and stable filament passing through the glass tube so steadily that it scarcely seemed to be in motion.

→ With increase in the velocity of flow a critical state was reached at which the filament of dye showed irregularities and began of waver.

→ With a further increase in the velo-city of flow the fluctuations in the filament of dye became more intense.

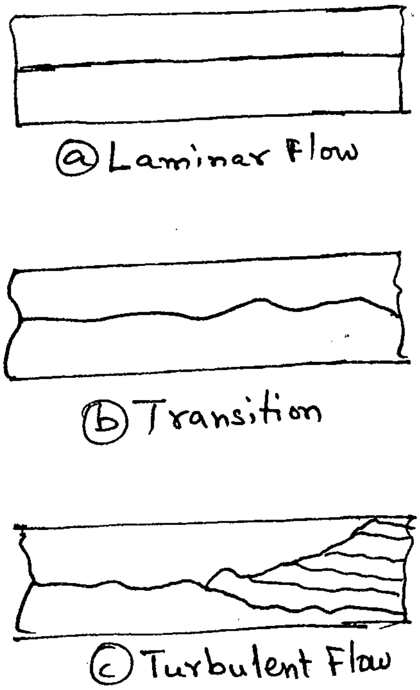


Fig: Appearance of dye filament in glass tube

→ Ultimately the dye diffused over the entire cross section of the tube, due to the intermingling of the particles of the flowing fluid.

→ Figure shows the different states of the dye filament.

→ Reynolds deduced from his experiments that at low velocities the intermingling of the fluid particles was altogether absent and that the fluid particles moved in parallel layers of laminae, sliding past adjacent laminae but not mixing with them. This is the regime of Laminar flow (figure (a)).

→ Since at higher velocities the dye filament diffused through the tube, it was apparent that the inter-mingling of the fluid particles was occuring, or in other words the flow was turbulent (figure (c)).

→ The velocity at which the flow changes from the laminar to turbulent for the case of a given fluid at a given temperature and in a given pipe is known as critical velocity.

→ The state of flow in between these two types of flow is known as 'transitional state' (or flow in transition).

On the basis of his experiments Reynolds discovered that the occurence of a laminar and turbulent flow was governed by the relative magnitudes of the inertia and the viscous forces.

→ It was indicated by Reynolds that at low velocities of flow even for the fluids having very small viscosity, the viscous forces become predominant and therefore the flow is largely viscous in character.

→ However, at higher velocities of flow the inertial forces have predominance over the viscous forces. Reynolds related the inertia to viscous forces and arrived at a dimensionless parameter.

$$Re \text{ or } N_R = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v}$$

→ According to Newton's second law of motion the inertia force F_i is given by

$$F_i = \text{mass} \times \text{acceleration}$$
$$= \rho \times \text{volume} \times \text{acceleration}.$$
$$= \rho \times L^3 \times (L/T^2) = \rho L^2 V^2$$

Similarly viscous force F_v is given by Newton's law of Viscosity as,

$$F_V = \tau \times \text{Area}$$
$$= \mu \frac{du}{dy} \times L^2 = (\mu V L)$$

$$Re \text{ or } N_R = \frac{(\rho L^2 V^2)}{\mu V L} = \frac{\rho V L}{\mu}.$$

→ This dimensionless parameter is called Reynold's number, in which ρ and μ are respectively the mass density and viscosity of the flowing fluid, V is the characteristic (or representative) velocity of flow and L is the characteristic linear dimension.

→ In the case of flow through pipes the characteristic linear dimension L is taken as the diameter D of the pipe and the characteristic velocity is taken as the average velocity V of flow of fluid.

→ Thus Reynold's ~~number~~ number becomes $\left(\frac{\rho D V}{\mu}\right)$ or $\left(\frac{VD}{\nu}\right)$ where $\left(\frac{\mu}{\rho}\right) = \nu$. Kinematic Viscosity of flowing fluid

→ The Reynold's number is therefore a very useful parameter in predicting whether the flow is laminar or turbulent.

→ The limiting values of Reynolds number corresponding to which the flow of fluid in a pipe is either laminar & turbulent are discussed later.

→ The existence of two flow regimes may also be indicated with the help of another simple experiment as shown in fig.

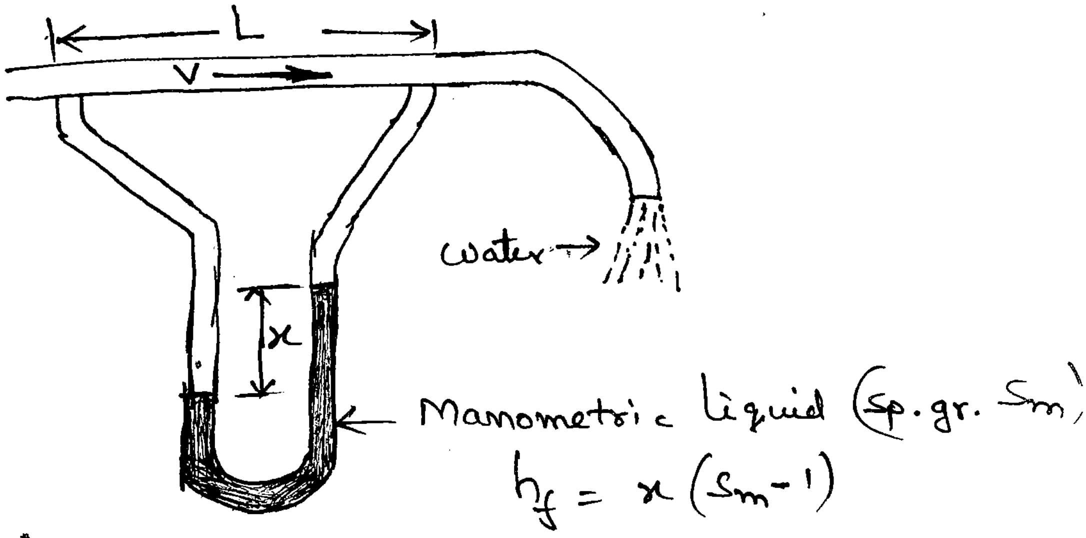


Fig: Apparatus for measuring the Loss of head in a pipe.

→ The apparatus required consists of a uniform horizontal pipe of a known diameter, to which a manometer is connected for measuring the loss of head h_f, occurring in a length L, of the pipe.

→ The head loss can be obtained from the manometer reading for a particular discharge and the mean velocity V of flow through pipe can be determined from the measured discharge.

→ Several values of the head loss can ~~be~~ thus be obtained for the corresponding values of the velocities of flow of fluid

(4)

→ Now if a logarithmic plot of (h_f/L) as ordinate and the velocity V as abscissa is prepared, it will be as shown in fig.

→ From this it will be found that for small values of 'V', the plot is a straight line with its slope equal to unity.

→ This continues upto certain value of V, represented by point B on the figure, which thus indicates that as long as the velocity is less than the value corresponding to point B,

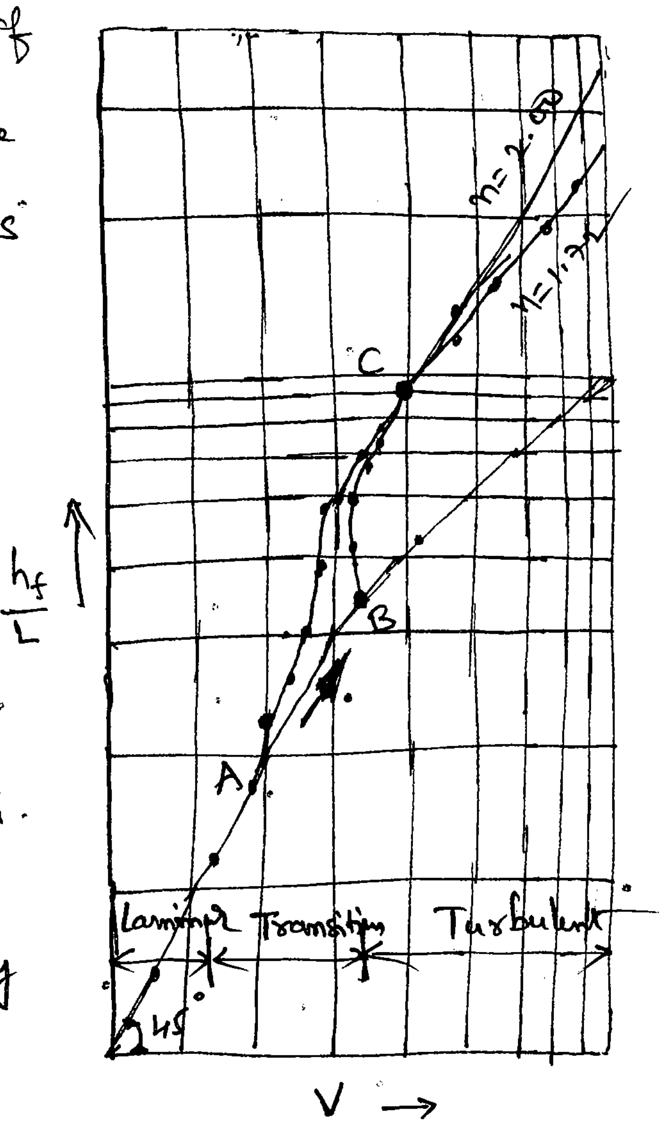


Fig. Plot of $\left(\frac{h_f}{L}\right)$ v/s V showing upper & lower critical points & velocities

the head loss due to friction will be directly proportional to the velocity of flow of fluid (i.e., $\frac{h_f}{L} \sim V$)

→ Beyond the point B with increasing velocity it will be found that there exists certain transition region extending upto point C.

→ During which there is an abrupt increase in the rate at which the loss of head varies.

→ After the region of transition has passed, again the curve obtained is in the form of straight lines with slopes ranging from 1.72 to 2.0

→ However, if the velocity is gradually reduced from a higher value, the line BC will not be retraced.

→ Instead the points may be along curve CA, as indicated by arrows in fig.

→ The point B is known as the upper critical point and the point A is known as the lower critical point

→ The corresponding velocities are known as upper critical velocity & lower critical velocity.

→ It is thus seen that upto point A, the drop in pressure head due to frictional resistance is directly proportional to the mean velocity of flow V, which is the range of laminar flow.

→ Beyond point C, the drop in pressure head due to frictional resistance varies as V_n, where n ranges from 1.72 to 2.0, which is the one of turbulent flow.

→ Between the points A and C (i.e., in between the regimes of laminar and turbulent flows) lies the transition region as shown in fig.

(5).

Conclusions from figure:

→ The upper critical Reynolds number corresponding to point B (i.e., upper limit of laminar flow in pipes) was found by Reynolds to lie between 12000 to 14000

→ But the upper critical Reynolds number is indefinite, being dependent upon initial disturbance affecting the flow, shape of entry of pipe, roughness of pipe wall etc

→ Thus the practical value of upper critical Reynolds number may be considered to lie between 2700 to 4000

→ The lower critical Reynolds number for flow of fluid in pipes corresponding to point A is of greater engineering importance

→ As it indicates a condition below which all turbulence entering the flow from any source will be damped out by viscosity and thus sets a limit below which laminar flow will always occur.

→ Experimentally the value of the lower critical Reynolds number has been found to be approximately 2000.

→ Between Reynolds numbers 2000 and 4000 the transition region exists.

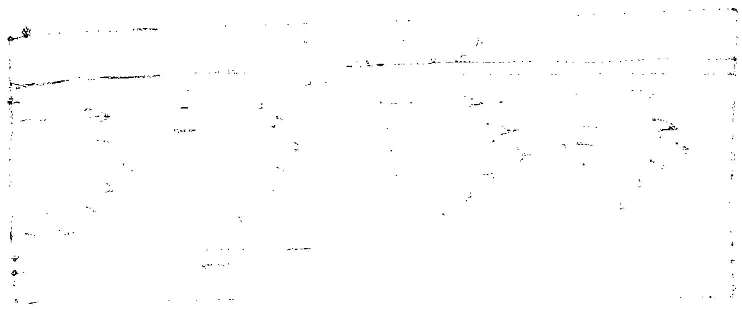
→ The concept of Critical Reynolds number which distinguishes the regimes of laminar and turbulent flow is indeed quite useful in the study of various fluid flow phenomena.

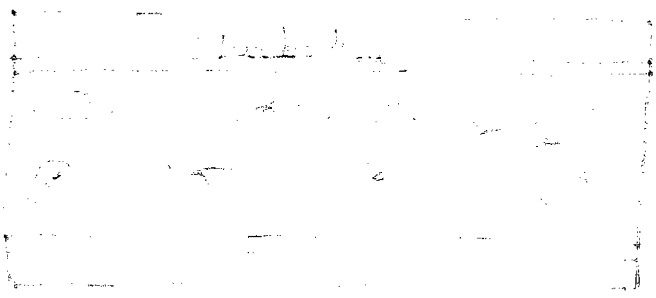
→ Applying this concept to the flow of any fluid in circular pipes, one may predict that the flow will be laminar if Reynolds number is less than 2000 and turbulent if it is greater than 4000.

→ It may however be pointed out that critical Reynolds number is very much a function of boundary geometry.

⇒ Characteristics of Laminar and Turbulent Flows.

Laminar Flow	Turbulent Flow
A Flow is said to be laminar when the various fluid particles move in layers (or laminae) with one layer of fluid sliding smoothly over an adjacent layer.	A fluid motion is said to be turbulent when the fluid particles move in an entirely haphazard or disorderly manner, that results in a rapid and continuous mixing of the fluid leading to momentum transfer as flow occurs.
It occurs at low velocities	It occurs at high velocities
Here there are no eddies nor vortices.	In such a flow eddies or vortices of different sizes & shapes are present which move over large distances.
When Reynolds number is less than 2000, the flow is laminar	When Reynolds number is greater than 4000, the flow is turbulent





(6)

Flow through Parallel Plates – Both Plates at Rest

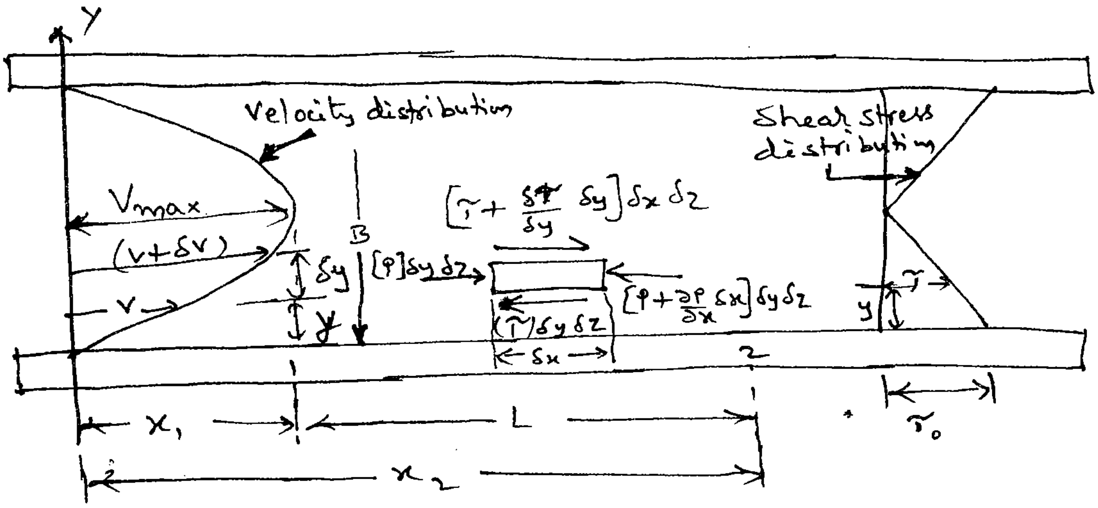


Fig: Laminar flow between two fixed parallel flat plates.

→ Consider laminar flow of fluid between two fixed parallel flat plates located at a distance B apart, as shown in figure

→ A small rectangular element of the fluid of length δx is as free-body as shown in figure.

→ Let the lower face of the element be at a distance y from the lower plate and here let the velocity be V.

→ At the upper face of the element which is at a distance of $(y+dy)$ from the lower plate, Let the velocity be $(v+\delta v)$

→ If δv is positive, the faster moving fluid just above the upper face of the element exerts a forward force across the upper face.

→ Similarly, the slower moving fluid adjacent to the lower face tends to retard its motion i.e., it exerts a backward force on the lower face.

→ Thus there are shear stresses of magnitude τ on the lower face and $(\tau + \frac{\partial \tau}{\partial y}\delta y)$ on the upper face of the element in the directions as shown in figure.

→ In order to balance the shearing forces in the fluid a pressure gradient in the direction of flow must be maintained.

→ Thus if p is the pressure intensity at the left face of the element, $\left(p + \frac{\partial p}{\partial x}\delta x\right)$ then will be the pressure intensity on the right face of the element.

→ If the width of the element in the direction perpendicular to the paper is δz, the total force acting on the element towards the right is

$$\left[p - \left(p + \frac{\partial p}{\partial x}\delta x\right)\right]\partial y\, \partial z + \left[\left(\tau + \frac{\partial \tau}{\partial y}\delta y\right) - \tau\right]\partial x\, \partial z.$$

→ But for steady and uniform flow, there is no acceleration and hence this total force must be equal to zero

$$-\frac{\partial p}{\partial x} \cdot \delta x\, \delta y \,.\delta z + \frac{\partial \tau}{\partial y}\delta x\, \delta y\, \delta z = 0$$

→ Dividing by the volume of the element $(\delta x\, \delta y\, \delta z)$, we get

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}.$$

→ Again according to Newton's Law of viscosity for laminar flow the shear stress $\tau = \mu \cdot \frac{du}{dy}$.

→ Hence by substitution in the above equation the following differential equation for laminar flow is obtained,

$$\frac{\partial p}{\partial x} = \mu \cdot \frac{\partial^2 v}{\partial y^2}.$$

→ Since, $\left(\frac{\partial p}{\partial x}\right)$ is independent of y, integrating the above equation twice with respect to y gives

$$v = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)\frac{y^2}{2} + C_1 y + C_2 \longrightarrow (a)$$

→ The two constants of integration c_1 and c_2 may be evaluated by means of two boundary conditions.

→ There being no slip of fluid at the two solid boundary surfaces of the plates,

$V = 0$ at $y = 0$

and $c_2 = 0$

$V = 0$ at $y = B$

→ Now we get the value of first constant of integration as $c_1 = -\frac{B}{2\mu}\left(\frac{\partial p}{\partial x}\right)$

→ Introducing these values in Equation (a) the following equation for the velocity distribution is obtained

$$V = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(By - y^2) \longrightarrow (b)$$

→ Equation (b) indicates that the velocity distribution curve for the laminar flow between parallel flat plate is a parabola with its vertex being mid-way between the plates.

→ The negative sign for the pressure gradient indicates that there is a drop in pressure in the direction of flow

→ The maximum velocity V_{max} which occur at the mid point between the plates,

$$V_{max} = \frac{B^2}{8\mu}\left(\frac{-\partial p}{\partial x}\right) \quad (or), \quad V = \frac{2}{3} V_{max}$$

→ Pressure gradient is given by, $\left(\frac{-\partial p}{\partial x}\right) = \frac{12\mu V}{B^2}$

→ Drop in pressure head (h_f) is given as,

$$h_f = \frac{P_1 - P_2}{w} = \frac{12 \mu V L}{w B^2}$$

→ The distribution of shear stress in the flowing fluid across any section of the parallel plates may be determined by substituting Equation (b) into Newton's Law of Viscosity

Thus, $\tau = \mu \frac{\partial v}{\partial y}$

& $\tau = \mu \frac{\partial}{\partial y}\left[\frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(By - y^2)\right]$

& $\tau = \left(-\frac{\partial p}{\partial x}\right)\left(\frac{B}{2} - y\right)$

* Flow through Long pipes:

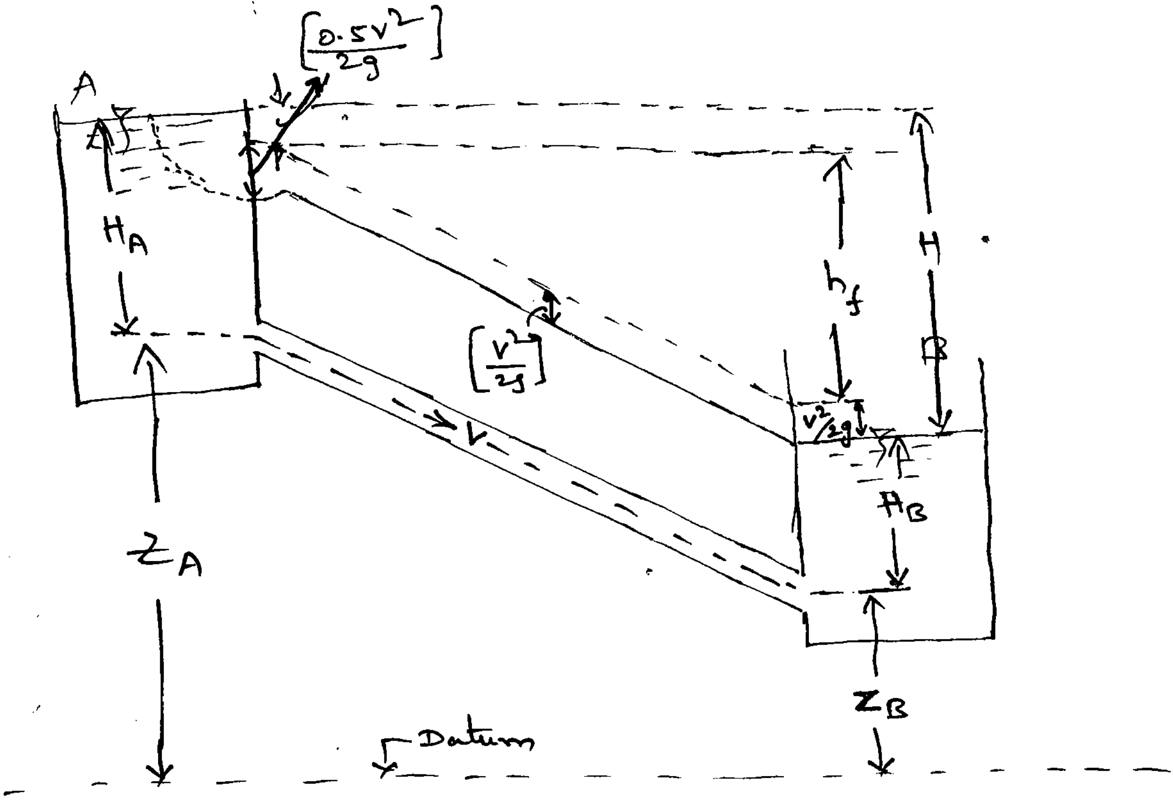


Fig: Flow through a long pipe

(8) → Consider a long pipe line of diameter D and length L carrying liquid from a reservoir A to another reservoir B, as shown in figure.

→ Let H_A and H_B be the constant heights of the liquid surfaces in the reservoirs A and B respectively above the centre of the pipe

→ Further let Z_A and Z_B be the heights of the centres of the pipe ends connected to the reservoirs A and B respectively.

→ Now if V is the mean velocity of flow through the pipe then the head loss due to friction (h_f),

$$h_f = \frac{fLV^2}{2gD}$$

where f - Friction factor
L - Length of Pipe (m).
V - Velocity of fluid flow (m/s),
g - Acceleration due to gravity (m/s²),
D - Diameter of pipe (in metres)

→ Head loss at the entrance of pipe, $= 0.5\frac{V^2}{2g}$

→ Head Loss at the exit of pipe, $= \frac{V^2}{2g}$

→ Applying Bernoulli's equation between points ① and ② in the reservoirs A and B respectively, we obtain

$$H_A + Z_A = H_B + Z_B + 0.5\frac{V^2}{2g} + \frac{fLV^2}{2gD} + \frac{V^2}{2g}$$

(or) $(H_A + Z_A) - (H_B + Z_B) = \frac{V^2}{2g}\left(1.5 + \frac{fL}{D}\right)$

But $(H_A + z_A) - (H_B + z_B) = H$

where H is the difference in the liquid surfaces in the reservoirs A and B. Thus

$$H = \frac{v^2}{2g}\left(1.5 + \frac{fL}{D}\right) \longrightarrow ①$$

→ Equation (1) indicates that the difference in the liquid surfaces in the two reservoirs at the two ends of the pipe is equal to the sum of the various head losses.

→ From this equation the unknown velocity may be computed

→ If the pipe is long (say, more than 1000 times the diameter) the loss of head due to friction will be very large as compared with the minor losses which may then be neglected, there by simplifying the expression as,

$$H = \frac{fLV^2}{2gD} \; ; \; \& \quad V = \sqrt{\frac{2gHD}{fL}}$$

→ If the pipe in figure, instead of discharging into the reservoir B, discharges into the atmosphere the equation would ~~be~~ then be,

$$(H_A + z_A) = z_B + 0.5\frac{v^2}{2g} + \frac{fLv^2}{2gD} + \frac{v^2}{2g}$$

$$\& \quad H = \frac{v^2}{2g}\left(1.5 + \frac{fL}{D}\right)$$

where H is the height of the liquid surface in the reservoir A above the outlet end of the pipe.

(9).

Flow through Inclined Pipes

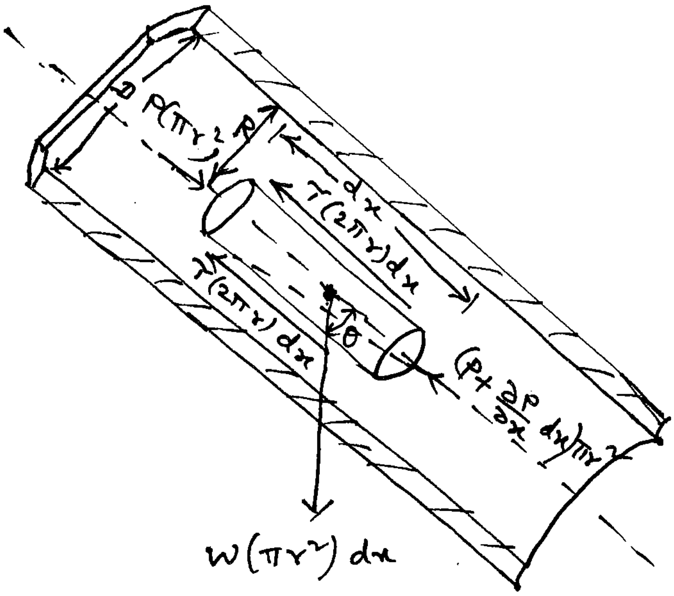


Fig: Laminar flow through an inclined pipe.

→ When the pipe having laminar flow of fluid is inclined as shown in figure in addition to pressure and viscous force gravity forces will also become effective.

→ Consider a small concentric cylindrical fluid element, if specific weight of the fluid is w, the weight of the element is $[w(\pi r^2)dx]$ acting vertically downward.

→ Its component in the direction of flow becomes $[w(\pi r^2)dx]\cos\theta$ in which x-direction is taken parallel to the axis of pipe

→ Now if z-direction is taken vertically upward, let dz represents a change in elevation corresponding to length dx of the fluid element.

→ Thus from figure, we get, $\cos\theta = -\frac{dz}{dx} = -\frac{\partial z}{\partial x}$.

→ In which the minus sign has been considered because the elevation z decreases as the distance x increases.

→ The component of the weight of the fluid element may thus be written as,

$$[w(\pi r^2)dx]\cos\theta = [w(\pi r^2)dx]\left(-\frac{dz}{dx}\right) = -w(\pi r^2)\,dz \rightarrow (1)$$

→ The other forces acting on the fluid element are the shear and pressure forces as shown in figure.

→ In the absence of any acceleration, sum of all the forces acting on the element in the direction of flow must be equal to zero. Thus

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - w\pi r^2 dz - \tau(2\pi r)dx = 0$$

$$\tau = -\frac{\partial}{\partial x}(P + wz)\frac{r}{2} = -\frac{\partial}{\partial x}\left(\frac{P}{w} + z\right)\frac{r}{2} \rightarrow (2)$$

$$\tau = -w\frac{\partial h}{\partial x}\frac{r}{2} \rightarrow (3)$$

→ In which, $h = \left[\frac{P}{w} + z\right]$ represents piezometric head.

→ For pipes, according to Newton's law of viscosity

$$\tau = -\mu\frac{\partial v}{\partial r}$$

→ We now have,

$$-\mu\frac{\partial v}{\partial r} = -w\frac{\partial h}{\partial x}\frac{r}{2}$$

(or), $$\frac{\partial v}{\partial r} = \frac{w}{\mu}\frac{\partial h}{\partial x}\frac{r}{2}$$

* The velocity profile for flow in an inclined pipe is given as,

$$\boxed{v = \frac{1}{4\mu} \times w \left(-\frac{\partial h}{\partial x}\right)\left(R^2 - r^2\right)}$$

Problems

1. A pipeline 0.225 m in diameter and 1580 m long has a slope of 1 in 200 for the first 790 m and 1 in 100 for the next 790 m. The pressure at the upper end of the pipeline is 107.91 kpa and at the lower end is 53.955 kpa. Taking f = 0.032 determine the discharge through the pipe.

Sol: Given Data:

Diameter of pipe (d) = 0.225 m

Length of pipe (L) = 1580 m.

Pressure at upper end (P_1) = 107.91 kpa

Pressure at lower end (P_2) = 53.955 kpa

Friction factor (f) = 0.032

Data to be calculated:

Discharge through pipe (Q)

Solution: Assuming the datum to be passing through the lower end of the pipe, the datum head for the upper end of the pipeline is

$$Z_1 = \frac{790}{200} + \frac{790}{100} = (3.95 + 7.90) = 11.85\,m.$$

Applying Bernoulli's equation between the upper and the lower ends of the pipeline,

$$\frac{P_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{v_2^2}{2g} + z_2 + h_f$$

But $V_1 = V_2 = V$

Then the loss of head due to friction,

$$h_f = \frac{fLV^2}{2gD} = \frac{0.032 \times 1580 \times V^2}{2 \times 9.81 \times 0.225} = 11.45 V^2$$

Thus by substitution, we get,

$$\frac{107.91 \times 10^3}{9810} + \frac{V^2}{2g} + 11.85 = \frac{53.955 \times 10^3}{9810} + \frac{V^2}{2g} + 0 + 11.45 V^2$$

or $11.45 V^2 = 17.35$

or $V = \sqrt{\frac{17.35}{11.45}} = 1.23 \text{ m/s}$

$\therefore$ Discharge, $Q = \frac{\pi D^2}{4} \times V$

$$= \frac{\pi (0.225)^2}{4} \times 1.23$$

$$= 0.0489 \text{ m}^3/\text{s}$$

Result:

Discharge through pipe $(Q) = 0.0489 \text{ m}^3/\text{s}$.

2. A 0.3m diameter pipe 2340 m long is connected with a reservoir whose surface is 72 m above the discharging end of the pipe. If for the last 1170 m, a second pipe of the same diameter be laid beside the first and connected to it, what would be the increase in the discharge? Take f = 0.02.

Given Data:

Diameter of Pipe (d) = 0.3 m

Length of pipe (L) = 2340 m

Elevation (or) head (H) = 72 m

Friction factor (f) = 0.02

Data to be calculated.

Increase in discharge $(Q_2 - Q_1)$

Solution:

We know that

$$H = \left(1.5 + \frac{fL}{D}\right)\frac{V^2}{2g}$$

Substituting the known values, & solving we get,

$$72 = \left(1.5 + \frac{0.02 \times 2340}{0.30}\right)\frac{V^2}{2 \times 9.81}$$

$$V = \sqrt{\frac{72 \times 2 \times 9.81}{157.5}} = 3 \text{ m/s}$$

Discharge is given by

$$Q_1 = \frac{\pi D^2}{4} \times V = \frac{\pi (0.30)^2}{4} \times 3 = 0.212 \text{ m}^3/\text{s}$$

In the second case Let Q_2 be the total discharge

Since for the second half of the length there are two parallel pipes of the same diameter, each pipe will carry discharge equal to $\left(\frac{Q_2}{2}\right)$

Also the velocity of flow in each pipe will be equal to half the velocity of flow V in the first half of the Length.

Thus applying Bernoulli's equation between the water surface in the reservoir and the outlet of pipe, we get,

$$72 = 0.5\frac{V^2}{2g} + \frac{0.02 \times 1170 \times V^2}{2g \times 0.30} * \frac{0.02 \times 1170 \times \left(\frac{V}{2}\right)^2}{2g \times 0.30} + \frac{\left(\frac{V}{2}\right)^2}{2g}$$

$$72 = \frac{V^2}{2g}\left[0.5 + 78.0 + 0.25\right]$$

$$V = V_2 = 3.79 \text{ m/s.}$$

$$\therefore Q_2 = \frac{\pi}{4}(0.30)^2 \times 3.79 = 0.268 \text{ m}^3/\text{s}$$

Increase in discharge $= Q_2 - Q_1$

$= 0.268 - 0.212$

$= 0.056 \text{ m}^3/\text{s}$

& Percentage increase in discharge $= \frac{0.056}{0.212} \times 100$

$= 26.42\%$.

Result: Increase in discharge $(Q_2 - Q_1) = 0.056$ m

3) Two parallel plates kept 0.1m apart have laminar flow of oil between them with a max. velocity of 1.5 m/s. Calculate the discharge per metre width, the shear stress at the plates, the difference in pressure in Pascal's between two points 20m apart, the velocity gradient at the plates and velocity at 0.02m from the plate take viscosity of oil to be 2.453 Ns/m².

Given Data:

Distance between parallel plates (d) = 0.1m

Maximum velocity (V_{max}) = 1.5 m/s

viscosity of oil (μ) = 2.453 Ns/m²

Data to be calculated:

Discharge per metre width (q)

Shear stress at plates (τ_0)

Difference in pressure ($P_1 - P_2$)

Velocity (V) at a distance 0.02m from plate.

Solution:

In this case the mean velocity of flow V is equal to two-thirds of the max. velocity

$$V = \frac{2}{3}(1.5) = 1.0 \text{ m/s}$$

The discharge q per metre width plate is given by

$$q = VB = (1.0 \times 0.1) = 0.1 \text{ m}^3/\text{s per m.}$$

we know that, $V = \frac{12 \mu V}{B^2} = \frac{12 \times 2.453 \times 1.0}{(0.1)^2} = 2943.6 \frac{N}{m^2}$ per

The shear stress at the plates is given by

$$\tau_0 = \left(-\frac{\partial p}{\partial x}\right)\frac{B}{2} = \frac{2943.6 \times 0.1}{2} = 147.18 \frac{N}{m^2}$$

The pressure difference between the two points is given by

$$(P_1 - P_2) = \frac{12 \mu V}{B^2} = \frac{12 \times 2.453 \times 1.0 \times 20}{(0.1)^2}$$

$$= 58872 \frac{N}{m^2} = 58.872 \frac{kN}{m^2}$$

The shear stress at plates is also given by

$$\tau_0 = \mu \left(\frac{\partial v}{\partial y}\right)_{y=0}$$

As such the velocity gradient at the plates is given by

$$\left(\frac{\partial v}{\partial y}\right)_{y=0} = \frac{\tau_0}{\mu} = \frac{147.18}{2.453} = 60\ s^{-1}.$$

The velocity v at a distance of 0.02 m from the plate is given by

$$v = \frac{1}{2\mu}\left(-\frac{\partial p}{\partial x}\right)(By - y^2)$$

$$= \frac{1}{2 \times 2.453}(2943.6)\left[(0.1 \times 0.02) - (0.02)^2\right]$$

$$= 0.96\ m/s$$

Result:

Discharge per metre width (q) = 0.3 m^3/s per turn
Shear stress at plates (τ_0) = 147.18 N/m^2
Difference in pressure ($P_1 - P_2$) = 58.872 kN/m^2
Velocity (v) at a distance 0.02 m from Plate = 0.96 m/s

Mr. P. Siva Sankar, M.Tech
Asst. Professor (Civil)

Lecture Handout

UNIT - IV

Closed Conduit Flow

Learning Objectives.

At the end of this topic, you will be able to:

→ Understand laws of fluid friction
→ Derive Darcy - Weisbach Equation
→ Explain Losses in pipe flow
→ Describe about pipes arranged in series & parallel
→ Draw the total energy line and hydraulic gradient line
→ Explain Moody's chart
→ solve problems based on formulae and concepts.

Learning outcomes:

By the end of this topic, you will be able to

Know the laws of fluid friction

Evaluate Darcy - weisbach equation

Understand the losses in pipe flow

know about pipes arranged in series and parallel

Illustrate total energy line and hydraulic gradient line

Understand Moody's chart

Evaluate problems based on formulae and concepts.

⟹ Laws of Fluid friction:-

→ The frictional resistance offered to the flow depends on the type of flow.

→ Different laws are obeyed by the frictional resistance in the laminar and turbulent flows.

→ On the basis of the experimental observations the laws of fluid friction for the two types of flows are

i, – Laws of fluid friction for laminar flow

ii, – Laws of fluid friction for Turbulent Flow.

→ The frictional resistance in laminar flow is as follows

Proportional to velocity of flow

⇓

Independent of Pressure

⇓

Proportional to area of surface in contact

⇓

Independent of nature of surface in contact

⇓

Greatly affected by variation of temperature on flowing fluid

⇒ Laws of fluid friction for Turbulent Flow

→ The frictional resistance in case of turbulent flow is as follows.

Proportional to $(\text{velocity})^n$, where n varies from 1.72 to 2.0

⇓

Independent of pressure

⇓

Proportional to density of flowing fluid & area of surface in contact

⇓

slightly affected by variation of temperature of flowing fluid

⇓

Dependent on nature of surface in contact

Froude's Experiments

→ W. Froude conducted a series of experiments to investigate frictional resistance offered to the flowing water by different surfaces.

Experimental setup:

→ The experiments were conducted in a tank about 100 m (300 ft) long, 11 m (36 ft) broad and 3 m (10 ft) deep containing water.

→ Thin wooden boards about 5 mm (3/16 in) thick, 0.475 m (19 in) wide and lengths varying from 0.6 m (2 ft) to 1.5 m (50 ft) were towed end wise in this tank by connecting them to a carriage running on rails provided on the sides of the tank.

→ The carriage was hauled along at speeds varying from 30 m (100 ft) to 300 m (1000 ft) per minute, by means of a wire rope passing around a drum.

→ The boards were towed in a completely submerged position such that the upper edge was about 0.45 m below the water surface in the tank and the force required to tow the board being measured.

→ In order to develop the surfaces of different types the surfaces of the boards were covered with varnish, tinfoil, calico and sand in turn.

Conclusions:

⇒ From the results of these experiments froude derived the following.

1. The frictional resistance varies approximately with the square of the body.

2. The frictional resistance varies with the nature of surface.

3. The frictional resistance per unit area of the surface decreases as the length of the board increases but is constant for long lengths.

→ Thus if f' is the frictional resistance per unit area of given surface at unit velocity, A is the area of wetted surface and V is the velocity, then the total friction resistance F is given by

$$F = f' A V^n.$$

→ Assuming the index, $n = 2$.

$$F = f' A V^2.$$

⇒ Equation for Head Loss in Pipes Due to Friction - Darcy - Weisbach Equation.

→ Consider a horizontal pipe of cross-sectional area A carrying a fluid with a mean velocity V.

→ Let 1 and 2 be the two sections of the pipe L distance apart, where let the intensities of pressure be P_1 and P_2 respectively.

→ By applying Bernoulli's equation between the sections 1 and 2, we obtain

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2 = V$ and $z_1 = z_2$

$$\text{Loss of head} = h_f = \frac{p_1}{w} - \frac{p_2}{w}$$

→ i.e., the pressure intensity will be reduced by the frictional resistance in the direction of flow.

→ The difference of pressure heads between any two sections is equal to the loss of head due to friction between these sections.

→ Let f' be the frictional resistance per unit area at unit velocity, then frictional resistance

$$= f' \times \text{Area} \times V^n$$

$$= f' \times PL \times V^n.$$

→ Where P is the wetted perimeter of the pipe.

→ The pressure forces at the sections 1 and 2 are $(p_1 A)$ and $(p_2 A)$ respectively. Thus resolving all the forces horizontally, we have

$$p_1 A = p_2 A + \text{frictional resistance}$$

$$\text{or } (p_1 - p_2) A = f' \times PL \times V^n$$

$$\text{or } (p_1 - p_2) = f' \times \frac{P}{A} \times L V^n$$

→ Dividing both sides by the specific weight w of the flowing fluid.

$$\frac{(P_1 - P_2)}{w} = \frac{f'}{w} \times \frac{P}{A} L V^n.$$

but $h_f = \frac{(P_1 - P_2)}{w}$, then

$$h_f = \frac{f'}{w} \times \frac{P}{A} L V^n.$$

→ The ratio of the cross sectional area of the flow (wetted area) to the perimeter in contact with the fluid (wetted perimeter) i.e $\left(\frac{A}{P}\right)$ is called Hydraulic Mean Depth (H.M.D) or hydraulic radius and is represented by m or R

→ Then, $h_f = \frac{f'}{w} \times \frac{L V^n}{m}$

→ For pipes running full, $m = \frac{A}{P} = \frac{\left(\frac{\pi D^2}{4}\right)}{\pi D} = \frac{D}{4}$

→ Substituting this in the equation for h_f and assuming $n = 2$

$\therefore$ we have, $h_f = \frac{4f'}{w} \cdot \frac{L V^2}{D}$

putting $\frac{4f'}{w} = \frac{f}{2g}$

$$h_f = \frac{f L V^2}{2g D} \quad \text{------} \; (1)$$

→ where f is known as friction factor, which is dimensionless quantity.

→ Equation 1 is known as Darcy-Weisbach equation which is commonly used for computing the loss of head due to friction in pipes.

→ It may be noted that the head loss due to friction is also expressed interms of the velocity head $\left(\frac{V^2}{2g}\right)$ corresponding to the mean velocity.